AD-A069 569   ADMIRALTY MARINE TECHNOLOGY ESTABLISHMENT TEDDINGTON--ETC   F/G 20/11
THE EFFECT OF ANALYSIS BANDWIDTH ON THE ACCURACY OF MEASUREMENT--ETC(U)
MAR 79   A W WALKER

UNCLASSIFIED   AMTE(N)TM79401   DRIC-BR-67702   NL

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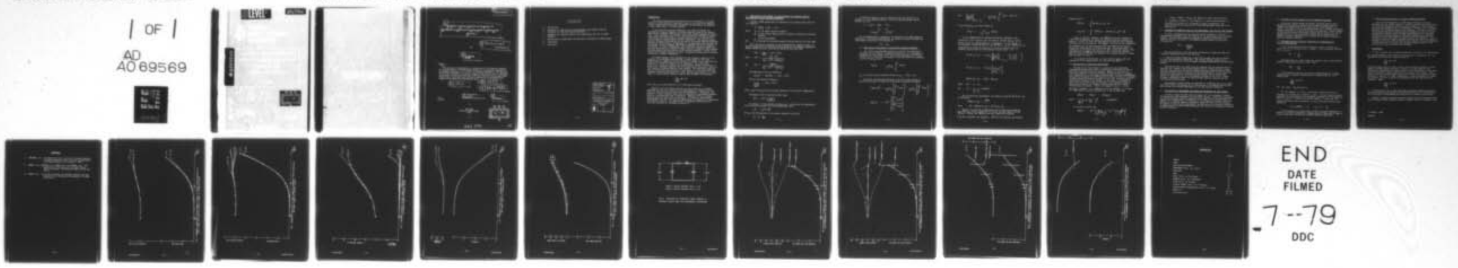

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(6) THE EFFECT OF ANALYSIS BANDWIDTH ON THE ACCURACY OF MEASUREMENTS AND PREDICTIONS OF SINGLE DEGREE OF FREEDOM SYSTEM BEHAVIOUR.

(9) Technical memo.

BY

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Summary

The accuracy of measurements or predictions of the frequency response of a single degree of freedom system depends on the frequency resolution of the analysis technique. If the ratio of analysis bandwidth to the 3dB bandwidth of the resonance is too great, negative bias errors occur at resonance in estimates of both power spectral levels and transfer function amplitudes. These errors, which have been calculated numerically, are presented and compared with appropriate experimental data.

(12) 24 p.

(18) DRIC

(19) BR-67702

AMTE (Teddington)
Queens Road
TEDDINGTON Middx TW11 OLN

(11) March 1979

26 pp
10 figs

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INTRODUCTION

The accurate measurement and prediction of the behaviour of a single degree of freedom system is important in any field in which the behaviour of more complex structures may be synthesised by superposing the behaviour of many single degree of freedom systems.

In the field of structural dynamics, it is usual to measure the experimental frequency response of a given system by using a dual channel Fast Fourier Transform (FFT) analyser with cross channel computation facilities. Such an analyser carries out a Discrete Fourier Transform on sampled time history data and outputs a frequency response which is defined at a finite number of equally spaced discrete frequencies. The spacing between adjacent frequency estimates may be defined as the nominal analysis bandwidth. Theoretical predictions of the behaviour of the same system may typically be made using the Finite Element (FE) method to generate the forced response of the system. In these circumstances, the theoretical frequency response is normally calculated at a series of chosen, discrete frequencies whose spacing may once again be defined locally as the analysis bandwidth. Values of the theoretical frequency response at other, intermediate frequencies are usually determined by linear interpolation.

In either analysis, full resolution of the frequency response is only possible if the analysis bandwidth Δf_a is small enough in the close neighbourhood of resonance in which the rate of change of both amplitude and phase of the frequency response will be maximised. For theoretical predictions, errors are clearly introduced whenever linear interpolation is applied to a frequency response that is insufficiently defined as a result of the frequency estimates being too widely spaced. For experimental measurements of broad band data, Forlifer (1) and Bendat et al (2) have shown that tolerable errors are introduced into power spectrum measurements of a single degree of freedom (SDOF) system as long as Δf_a satisfies the criterion,

$$\frac{\Delta f_a}{\Delta f_s} \leqslant 0.25$$

where Δf_s is the 3dB bandwidth of the SDOF system resonance.

However, as will be shown in Section 2, the errors due to analysis bandwidth are much more significant for measurements of transfer function amplitude than they are for power spectrum measurements. Fortunately, the presence of these increased errors will be indicated in experimental measurements by a corresponding loss of coherence. Expected errors in transfer function phase are also shown in Section 2. In Section 3, experimental measurements taken from an electrical SDOF system are presented to substantiate the numerical results. The implications of bandwidth induced errors to structural analysis are discussed in Section 4.

2. DERIVATION OF BIAS ERRORS IN POWER SPECTRUM AND TRANSFER FUNCTION ESTIMATES FOR VARYING ANALYSIS BANDWIDTHS.

Consider a SDOF system which is described by the second order differential equation

$$\ddot{y} + 2\zeta w_N \dot{y} + w_N^2 y = x(t)$$

where w_N is the system natural frequency

ζ is the viscous damping factor, stated as a fraction of critical damping

y is the displacement

and $x(t)$ describes an externally applied forcing function, per unit mass.

$Y(w)$, the Fourier transform of the displacement is related to $X(w)$, the Fourier transform of the forcing function by way of the frequency response or system transfer function which may be defined as $H(w)$ such that

$$H(w) = \frac{Y(w)}{X(w)} = a(w) + ib(w)$$

where
$$a(w) = \frac{(w_N^2 - w^2)}{(w_N^2 - w^2)^2 + 4\zeta^2 w^2 w_N^2}$$

and
$$b(w) = \frac{-2\zeta w w_N}{(w_N^2 - w^2)^2 + 4\zeta^2 w^2 w_N^2}$$

The magnitude of $H(w)$ is defined by

$$|H(w)|^2 = H(w)H^*(w) = a^2(w) + b^2(w)$$

Also at the discrete frequency w

$$\frac{G_{yy}(w)}{G_{xx}(w)} = a^2(w) + b^2(w)$$

where $G_{xx}(w)$ and $G_{yy}(w)$ are the power spectra of $x(t)$ and $y(t)$ respectively.

The phase of $H(w)$ is defined by $\emptyset(w)$ where

$$\emptyset(w) = \tan^{-1}\left(\frac{b(w)}{a(w)}\right)$$

In addition, if the resonant frequency w_R, is defined as the frequency at which the forced response has maximum amplitude, then

$$w_R^2 = w_N^2(1-2\zeta^2)$$

and w_S, the 3dB bandwidth of the system resonance is given by

$$w_S \doteqdot 2\zeta w_N$$

If hysteretic damping, which is defined by the loss factor η, is assumed to be the damping mechanism, then the foregoing argument may be modified to yield

$$\ddot{y} + w_N^2(1 + i\eta)y = x(t)$$

$$w_R = w_N$$

$$|H(w_R)|^2 = \eta^{-2} w_N^{-4}$$

In the neighbourhood of resonance, the response of the SDOF system is insensitive to the type of damping that is assumed as long as the numerical relationship between ζ and η is defined as

$$\eta = 2\zeta$$

2.1 Bias errors attributable to experimental analysis bandwidth.

If a finite bandwidth FFT analyser is used for the experimental estimation of the frequency response function $H(w)$, it is important to recognise that when using a broad band forcing function, the estimate of $H(w)$ that is output by the analyser in the band defined by $w_1 \leq w \leq w_2$ is given by $H_M(w_{12})$ where

$$H_M(w_{12}) = \frac{1}{w_2 - w_1} \int_{w_1}^{w_2} H(w)dw$$

w_{12} is the band centre frequency defined by $w_{12} = \frac{1}{2}(w_1 + w_2)$.

Similarly, experimental estimates of the other functions may be defined as follows, the subscript M denoting the measured estimates

$$|H_M(w_{12})|^2 = \frac{1}{w_2 - w_1}\left[\int_{w_1}^{w_2} a(w)dw\right]^2 + \frac{1}{w_2 - w_1}\left[\int_{w_1}^{w_2} b(w)dw\right]^2$$

$$\phi_M(w_{12}) = \tan^{-1}\left[\frac{\int_{w_1}^{w_2} b(w)dw}{\int_{w_1}^{w_2} a(w)dw}\right]$$

and $$\left[\frac{G_{yy}(w)}{G_{xx}(w)}\right]_{M,w = w_{12}} = \frac{1}{(w_2 - w_1)} \int_{w_1}^{w_2} a^2(w) + b^2(w)\,dw.$$

$H(w_{12})$ and $H_M(w_{12})$ are also related by

$$H(w_{12}) = \lim_{(w_2 - w_1)\to 0} H_M(w_{12})$$

In the neighbourhood of resonance defined by $w_1 \leqslant w_R \leqslant w_2$, full resolution of $H(w)$ requires that the measured estimate $H_M(w_{12})$ should define closely the system behaviour at resonance. In this context, it is convenient to define $\Delta P(\Delta w, \epsilon, w_R)$, $\Delta H(\Delta w, \epsilon, w_R)$ and $\Delta \phi(\Delta w, \epsilon, w_R)$ respectively as errors at resonance in the power spectrum amplitude, transfer function amplitude and transfer function phase. Thus

$$\Delta P(\Delta w, \epsilon, w_R) = 10\log_{10}\left\{\left[\frac{G_{yy}(w)}{G_{xx}(w)}\right]_{M,w=w_{12}} \cdot \left[\frac{G_{yy}(w_R)}{G_{xx}(w_R)}\right]^{-1}\right\}$$

$$\Delta H(\Delta w, \epsilon, w_R) = 10\log_{10}\left[\frac{|H_M(w_{12})|^2}{|H(w_R)|^2}\right]$$

$$\Delta \phi(\Delta w, \epsilon, w_R) = \phi(w_R) - \phi_M(w_{12})$$

where $\Delta w = w_2 - w_1$

and $\epsilon = \dfrac{w_R - w_1}{w_2 - w_1} \quad (0 \leqslant \epsilon \leqslant 1)$

For any particular measurement, the expected coherence $\gamma^2(\Delta w, \epsilon, w_R)$ is defined by

$$\gamma^2(\Delta w, \epsilon, w_R) = 10^{\beta/10}$$

where $\beta = \Delta H(\Delta w, \epsilon, w_R) - \Delta P(\Delta w, \epsilon, w_R)$

However, in practical circumstances, ϵ is effectively random and as a result it is more convenient to utilise the mean expected errors $\overline{\Delta P}(\Delta w)$, $\overline{\Delta H}(\Delta w)$ and $\overline{\Delta \phi}(\Delta w)$ and their respective variances $\sigma^2_p(\Delta w)$, $\sigma^2_H(\Delta w)$ and $\sigma^2_\phi(\Delta w)$. $\overline{\Delta P}(\Delta w)$ and $\sigma^2_p(\Delta w)$ are defined

respectively by

$$\overline{\Delta P}(\Delta w) = \int_0^1 \Delta P(\Delta w, \epsilon, w_R) \, d\epsilon$$

$$\sigma^2_p(\Delta w) = \int_0^1 (\overline{\Delta P}(\Delta w) - \Delta P(\Delta w, \epsilon, w_R))^2 \, d\epsilon$$

Values of $\overline{\Delta P}(\Delta w)$, $\overline{\Delta H}(\Delta w)$ and $\overline{\Delta \phi}(\Delta w)$ have been evaluated on a PDP 11/40 minicomputer by numerical integration and are presented in Figures 1, 2 and 3 in which the mean expected errors and their standard deviations are plotted against $(\Delta w/2\zeta w_N)$. To carry out the integration the bandwidth Δw has been subdivided into 200 bands and 500 evenly distributed random values of ϵ have been used. Using these data, the integration is convergent and accurate to four significant figures. The same numerical integration has also been used to calculate the mean expected coherence $\overline{\gamma}^2(\Delta w)$ and values of its standard deviation. These data are shown in Figure 4.

It is apparent from Figures 1, 2 and 3 that the mean errors are insensitive to the damping factor ζ, provided that $\zeta \leq 0.1$.

2.2 Bias errors in theoretical predictions.

The mean errors which affect the results of finite bandwidth experimental analysis also apply to theoretical predictions in which a broad band forcing function is utilised. However, in the field of structural dynamics, a discrete frequency forcing function is normally used, especially for finite element calculations. In these circumstances, the predicted response of a SDOF system will differ from the true resonant response unless the discrete frequency of the input is exactly equal to the resonant frequency of the system. In this case, it is simple to define the errors which occur in estimates of the resonant values of power spectrum amplitude, transfer function amplitude and transfer function phase as $\Delta P_T(w)$, $\Delta H_T(w)$ and $\Delta \phi_T(w)$ where w is the excitation frequency. If a sinusoidal force of frequency w is input to a SDOF system of natural frequency w_N, it may be shown that

$$\Delta H_T(w) = \Delta P_T(w) = 10\log_{10}\alpha$$

where $$\frac{w - w_N}{\zeta w_N} \doteqdot \pm\left(\frac{1}{\alpha} - 1\right)^{\frac{1}{2}} \qquad (0.25 \leq \alpha \leq 1)$$

It may also be shown that

$$\Delta\phi_T(w) = \tan^{-1}\left(\frac{2\zeta w w_N}{w^2 - w_N^2}\right) + \tan^{-1}\left(\frac{1 - 2\zeta^2}{\zeta^2}\right)^{\frac{1}{2}}$$

Values of $\Delta H_T(w)$, $\Delta P_T(w)$ and $\Delta\phi_T(w)$ are shown plotted against $(w - w_N)/\zeta w_N$ in Figure 5. From Figure 5, it may be determined that if the Forlifer criterion (1) is adopted in these circumstances, then amplitude errors are restricted to 0.3dB although phase errors may be as great as 14°.

3. COMPARISON OF NUMERICAL RESULTS WITH EXPERIMENTAL DATA FOR TWO SDOF SYSTEMS.

In order to substantiate the numerical results presented in Section 2, experimental measurements have been carried out on two separate electrical SDOF systems. Each of these resonators comprised a resistance, inductance and capacitor as shown in Figure 6. The measured system transfer function H(w) is defined as

$$H(w) = \frac{V_{OUT}(w)}{V_{IN}(w)}$$

The two resonators, which had natural frequencies of 500Hz and 4kHz, had respective damping factors of 0.25 and 0.05.

A Spectral Dynamics SD360 dual channel FFT processor was used to measure H(w) for varying analysis bandwidths. The nominal analysis bandwidth was increased by a factor of 1.8 by the SD360's standard Kaiser-Bessel data smoothing window.

Experimental data from these tests are compared with the numerical data in Figures 7-10. Figures 7 and 8 show respectively the transfer function errors that were obtained using the 4kHz and 500Hz resonators, whereas Figure 9 shows the power spectrum errors which were derived simultaneously. In all cases, the theoretical mean errors are indicated by solid lines, and the theoretical scatter is represented by twice the standard deviation of the error. Figure 10 shows a comparison between the predicted mean expected coherence and the experimentally derived data.

The agreement between theory and experiment is remarkably good in all cases. Any slight errors are probably due to inaccuracies in the damping factors which were derived from optimum measurements of the 3dB resonance bandwidth.

4. APPLICATIONS TO MEASUREMENT AND PREDICTIVE PROCEDURES FOR SDOF SYSTEMS.

In order to apply the data shown in Figures 1-5 it is necessary to define the errors which are acceptable in a particular analysis. Given this information, the maximum tolerable analysis bandwidth may be read directly from whichever of Figures 1-5 is appropriate. In addition it is necessary to consider three other features of structural analysis techniques, namely the effect of data windows as applied in digital signal processing, the application of frequency translation to measurements of lightly damped systems and the non-uniform accuracy of constant bandwidth analysis.

4.1 The effect of data windows in finite bandwidth analysis.

In general, digital signal processing techniques require that a smoothing window should be applied to all types of signals with the exception of signals that may be classified as being "transient" in nature. The effect of such a smoothing window is to increase the nominal analysis bandwidth by a factor which may be as much as 2. Further details of the precise effects of particular windows may be found in (3), but in general, the use of any data window produces greater inaccuracies in measured frequency response data as a result of the implied increase in effective analysis bandwidth.

4.2 The application of frequency translation to measurements of lightly damped systems.

If an N point Discrete Fourier Transform is used to process time series data in the frequency range $0 \leqslant f \leqslant f_o$, then the nominal bandwidth Δf is given by

$$\Delta f = \frac{2f_o}{N}$$

The application of a data window may, however, lead to the effective bandwidth Δf_a being as much as $2\Delta f$. Thus

$$\Delta f_a = \frac{4f_o}{N}$$

If the Forlifer criterion is applied to measurements of a lightly damped system of resonant frequency f_R and damping factor ζ, it is required that

$$\frac{\Delta f_a}{2\zeta f_R} \leqslant 0.25$$

For $\zeta = 0.001, \quad \Delta f_a \leqslant 0.0005\, f_R$

If, for example, $f_R = 0.5 f_o$, the Forlifer condition may only be satisfied if a 20,000 point transform is used. If such a facility is not available the only solution is to translate frequencies in the neighbourhood of f_R down to a baseband defined by $0 \leqslant f \leqslant f_o$. In this way, the required resolution may be achieved, but only in the much narrower band defined by

$$f_R - f_a \leqslant f \leqslant f_R + f_b \qquad (f_a + f_b = f_o)$$

If N is subject to an upper limit of 1024, frequency translation is likely to be required whenever the damping factor is less than or equal to 0.02, in order that the Forlifer criterion should be satisfied.

4.3 The non-uniform accuracy of constant bandwidth analysis.

Since the 3dB bandwidth of a SDOF system resonance increases linearly with the resonant frequency, constant accuracy can be maintained over the whole frequency range of a multiple degree of freedom system, for which the damping factor is independent of frequency, if the analysis bandwidth also increases linearly with frequency. This may be achieved by utilising constant percentage bandwidth analysis in which the centre frequencies of adjacent bands increase logarithmically with frequency. For discrete frequency analysis, consistent accuracy may be similarly achieved by using logarithmically spaced excitation frequencies. The variable accuracy of constant bandwidth analysis can thus be an important drawback to the use of FFT processing since accuracy at the low frequencies is sacrificed with the effect that the high frequency portion of the spectrum can become needlessly over-resolved.

5. CONCLUSIONS

If Δf_a is defined as the analysis bandwidth, and Δf_s as the 3dB bandwidth of a system resonance, then from the preceding text it is apparent that the application of the Forlifer condition given by

$$\frac{\Delta f_a}{\Delta f_s} \leq 0.25$$

produces minimal errors of order 0.1dB in power spectral estimates of single degree of freedom system behaviour. However, it is also apparent that simultaneous measurements of transfer function data are subject to much greater errors of the order 0.3dB in amplitude and as much as 14° in phase. These phase errors may be particularly important if further data reduction by modal analysis is involved. In these circumstances, it may be necessary to tighten the Forlifer condition to derive six estimates within the 3dB structural bandwidth. Thus

$$\frac{\Delta f_a}{\Delta f_s} \leq 0.17$$

In addition the use of data smoothing windows and frequency translation have been identified as important contributory factors in the determination of the accuracy of measurements and predictions of transfer function data.

Finally, constant percentage bandwidth analysis may be recommended since this technique retains constant accuracy throughout the whole frequency range.

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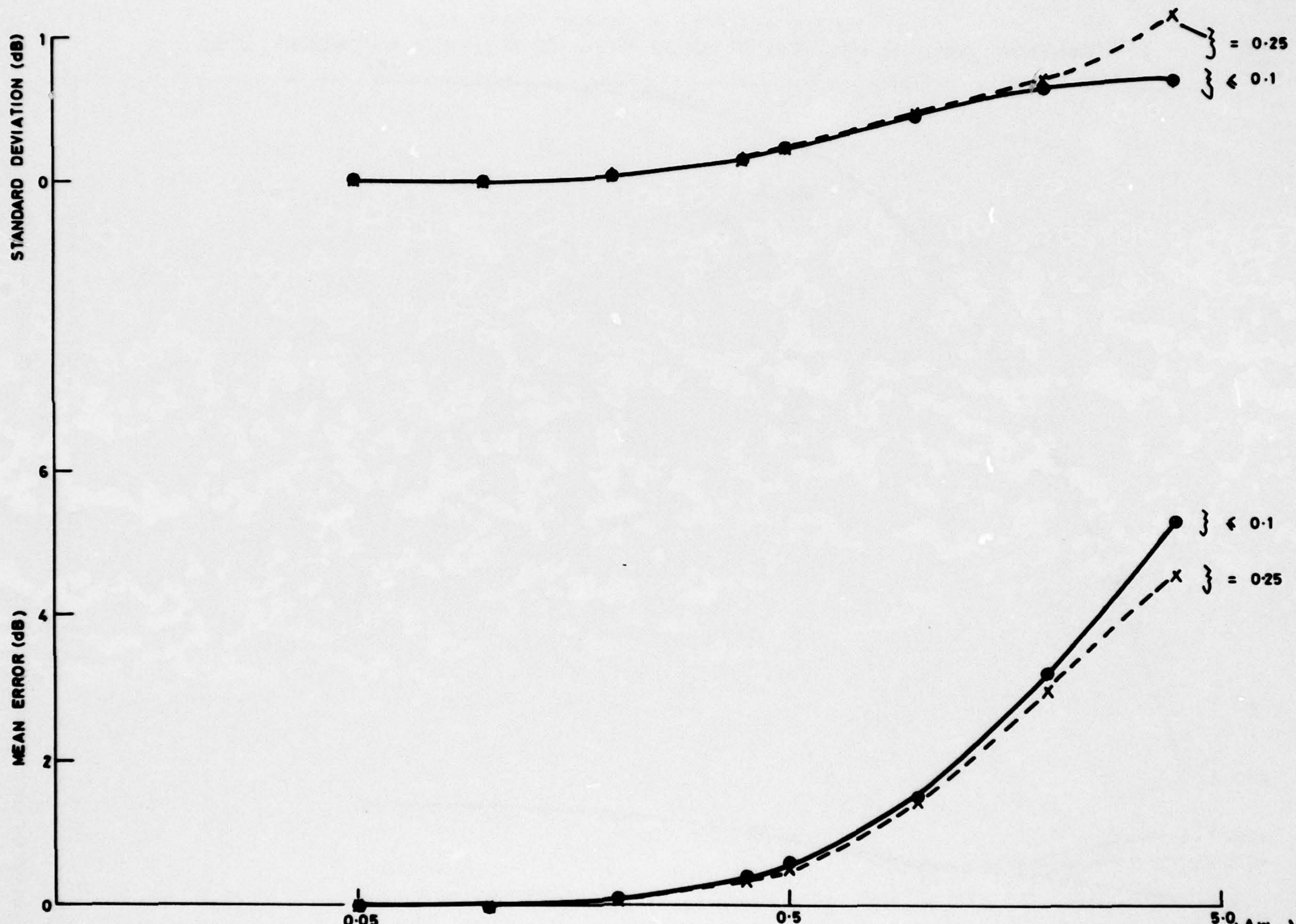


FIG. 1 BANDWIDTH VARIATION OF MEAN ERROR IN RATIO OF OUTPUT POWER SPECTRUM TO INPUT POWER SPECTRUM FOR SINGLE DEGREE OF FREEDOM SYSTEM

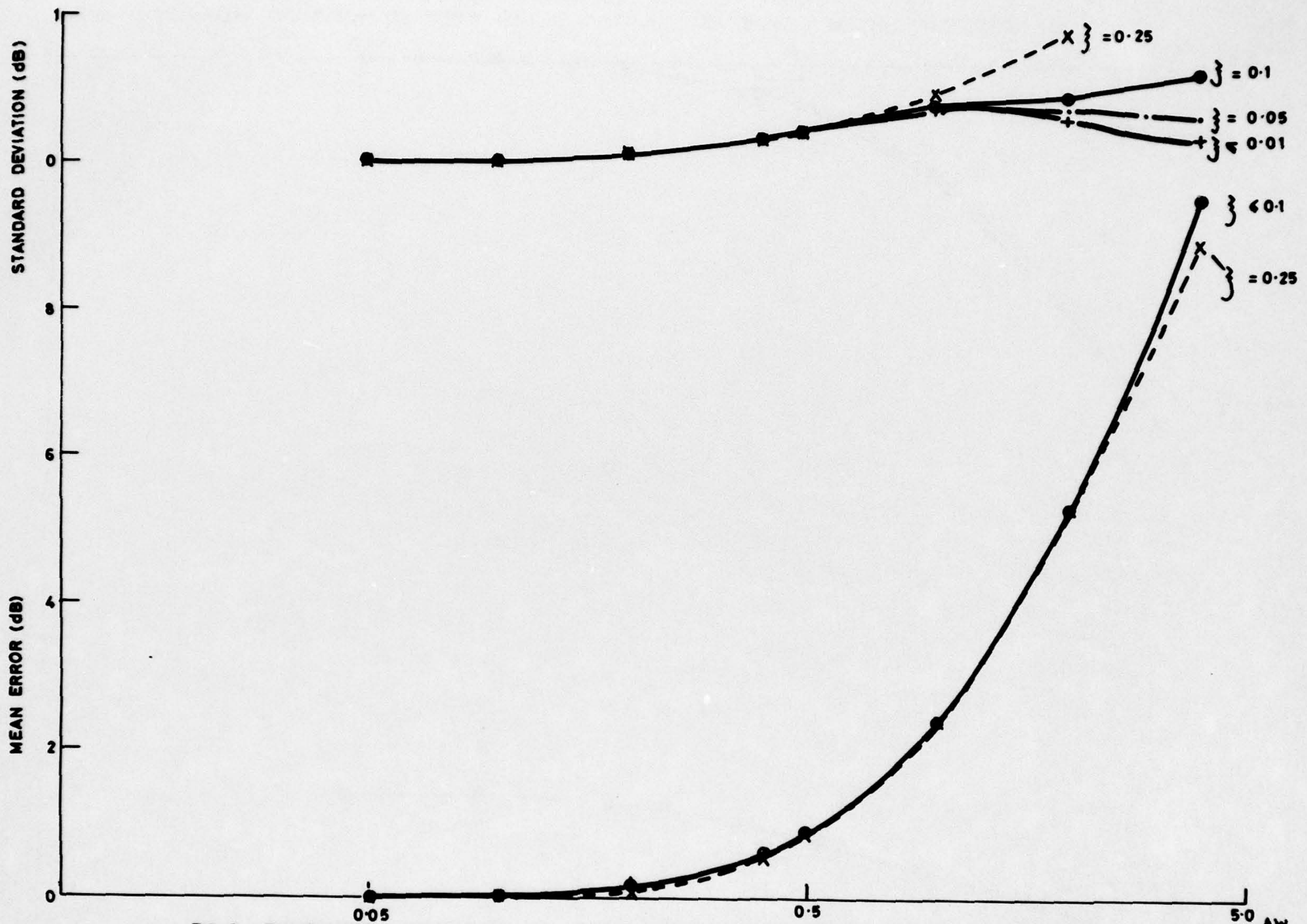


FIG. 2 BANDWIDTH VARIATION OF MEAN ERROR IN TRANSFER FUNCTION AMPLITUDE FOR SINGLE DEGREE OF FREEDOM SYSTEM

FIG. 3 BANDWIDTH VARIATION OF MEAN ERROR IN TRANSFER FUNCTION PHASE FOR SINGLE DEGREE OF FREEDOM SYSTEM

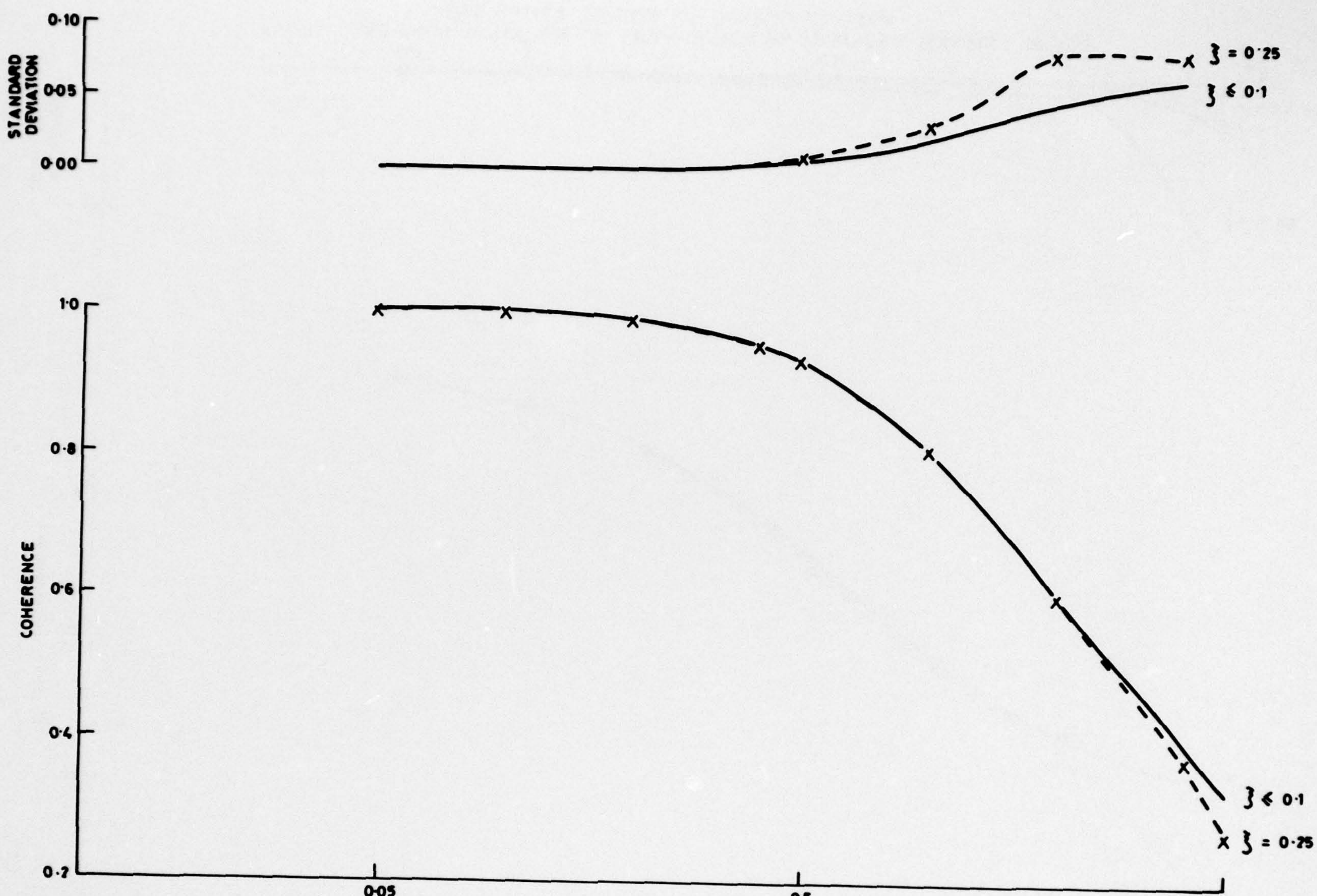


FIG. 4 BANDWIDTH VARIATION OF TRANSFER FUNCTION COHERENCE FOR SINGLE DEGREE OF FREEDOM SYSTEM

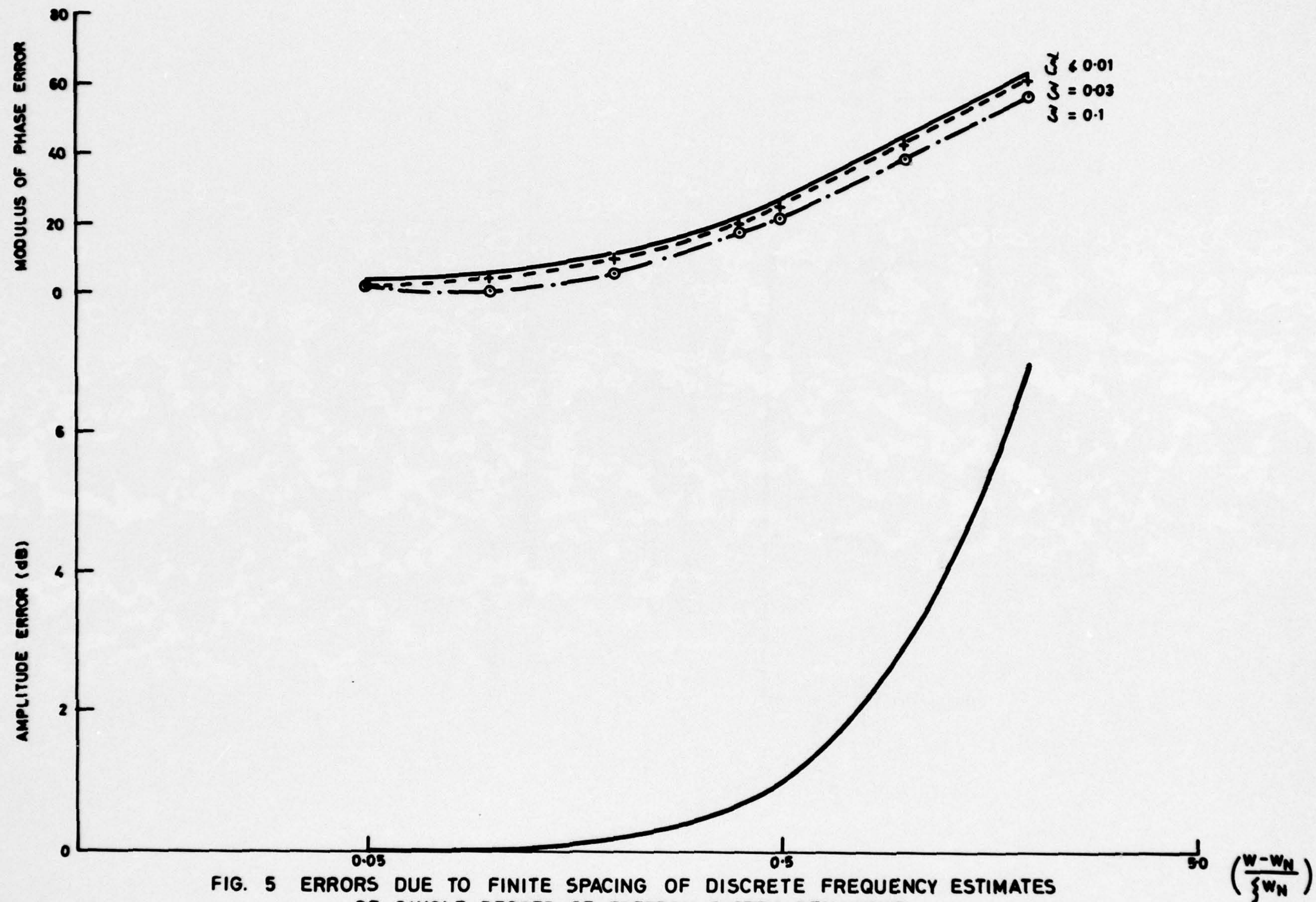
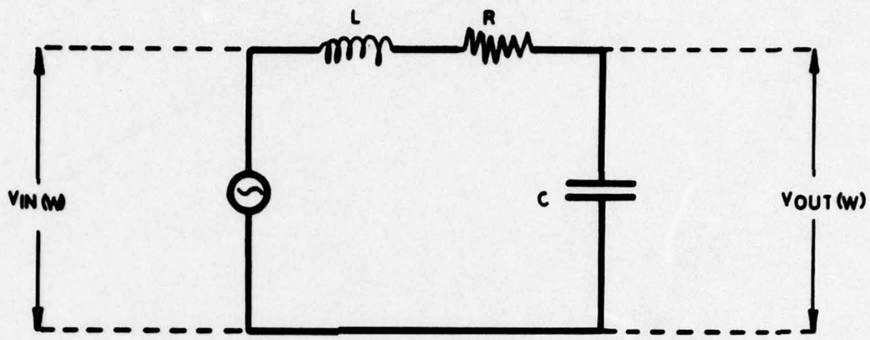


FIG. 5 ERRORS DUE TO FINITE SPACING OF DISCRETE FREQUENCY ESTIMATES OF SINGLE DEGREE OF FREEDOM SYSTEM BEHAVIOUR



SYSTEM 1 = NATURAL FREQUENCY 500Hz, ζ = 0·25

SYSTEM 2 = NATURAL FREQUENCY 4kHz, ζ = 0·05

FIG. 6 DEFINITION OF ELECTRICAL SINGLE DEGREE OF FREEDOM SYSTEM USED FOR EXPERIMENTAL PROGRAMME

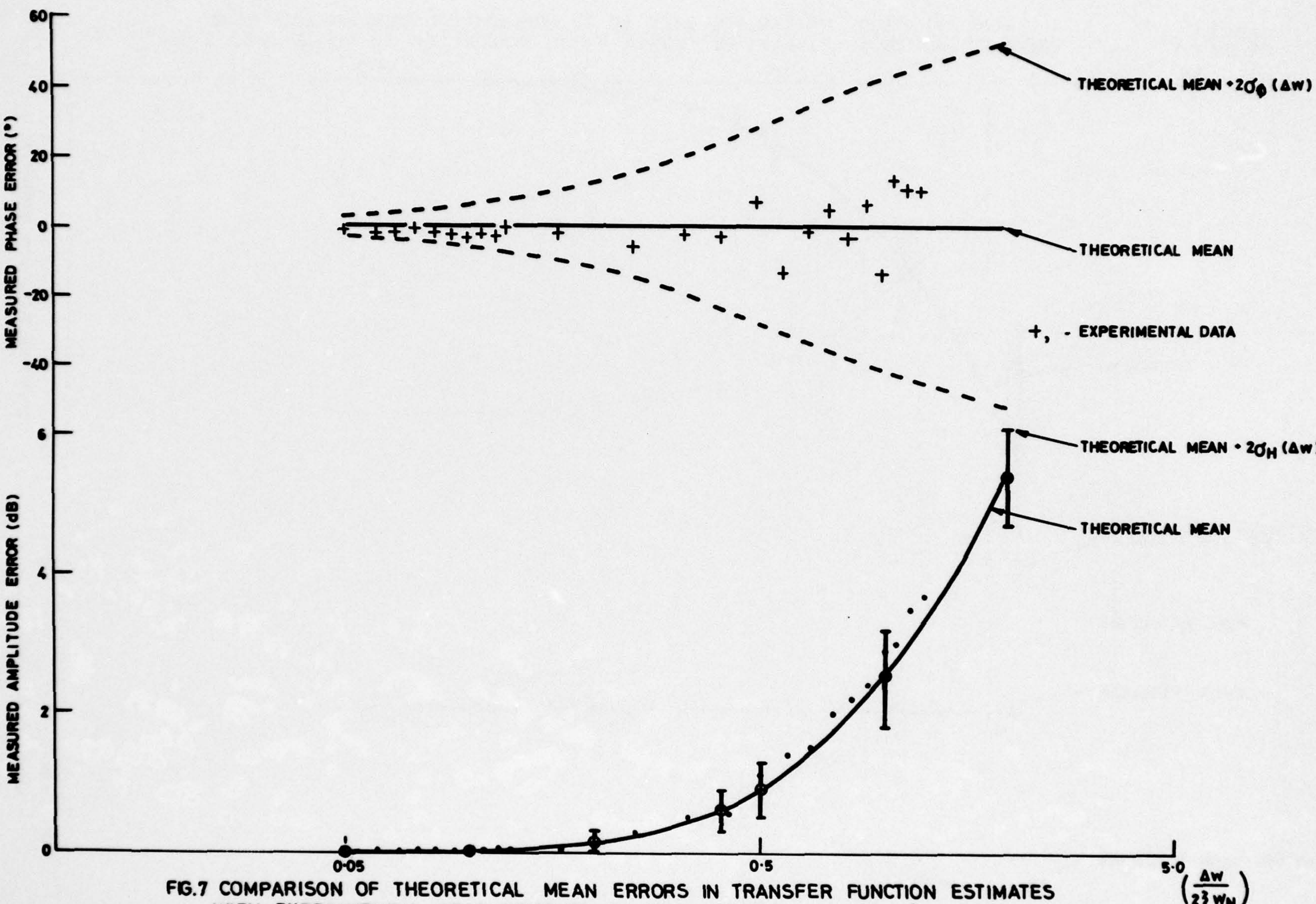


FIG.7 COMPARISON OF THEORETICAL MEAN ERRORS IN TRANSFER FUNCTION ESTIMATES WITH EXPERIMENTAL MEASUREMENTS OF SDOF SYSTEM, $f_N = 4$kHz, $\zeta = 0{\cdot}05$

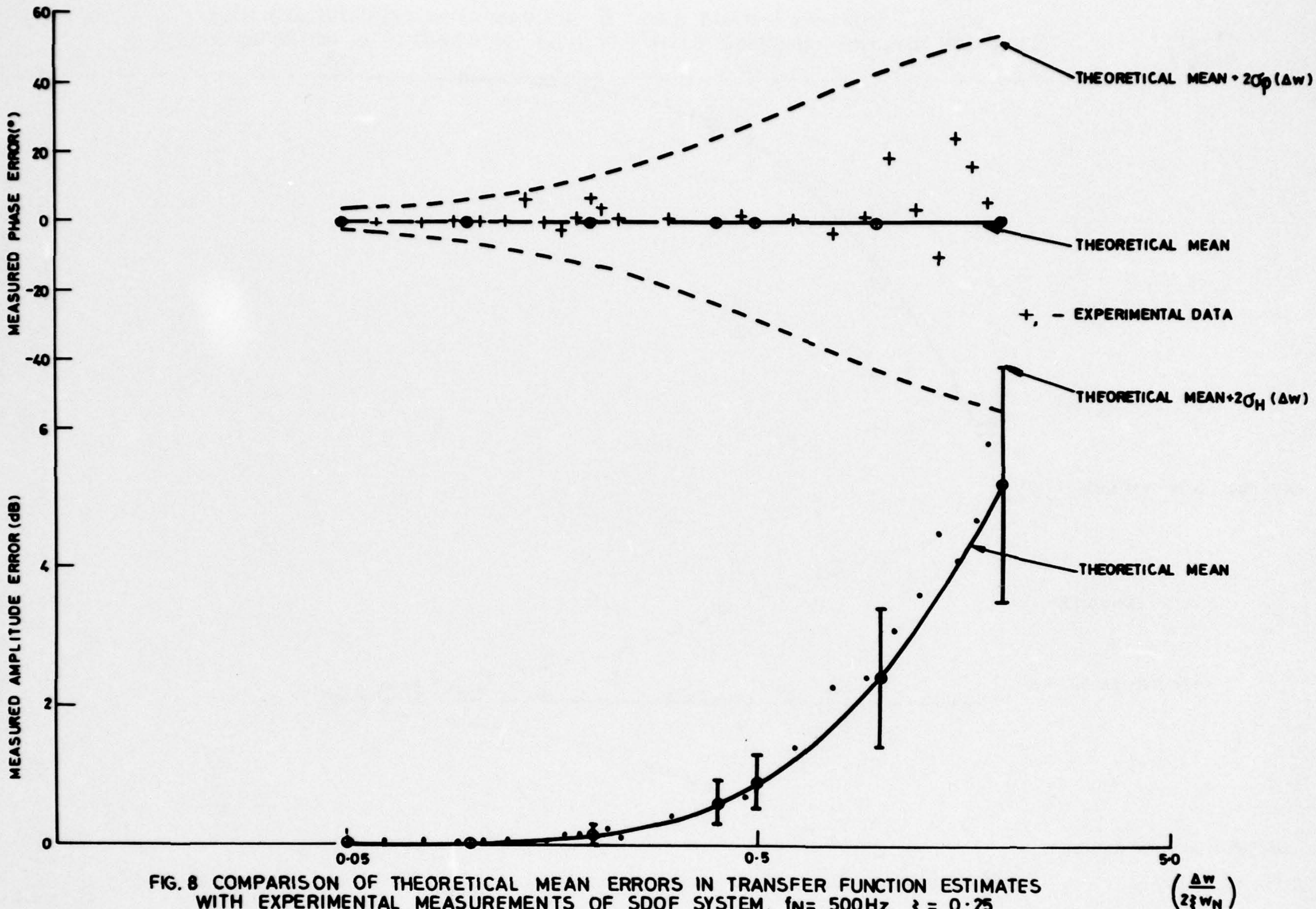


FIG. 8 COMPARISON OF THEORETICAL MEAN ERRORS IN TRANSFER FUNCTION ESTIMATES WITH EXPERIMENTAL MEASUREMENTS OF SDOF SYSTEM, f_N = 500 Hz, ζ = 0·25

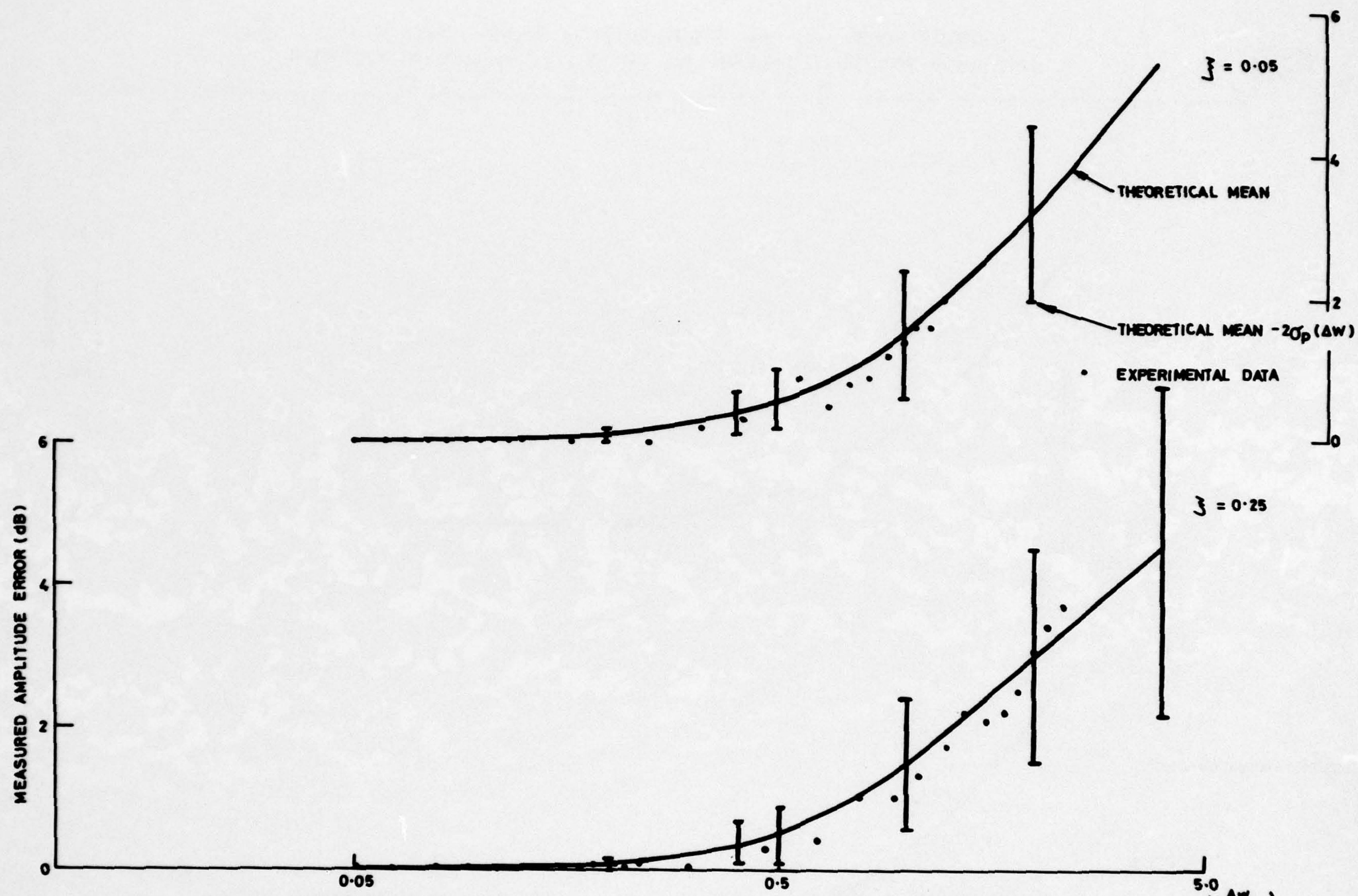


FIG. 9 COMPARISON OF THEORETICAL MEAN ERRORS IN POWER SPECTRUM RATIO WITH EXPERIMENTAL MEASUREMENTS OF BEHAVIOUR OF TWO SDOF SYSTEMS.

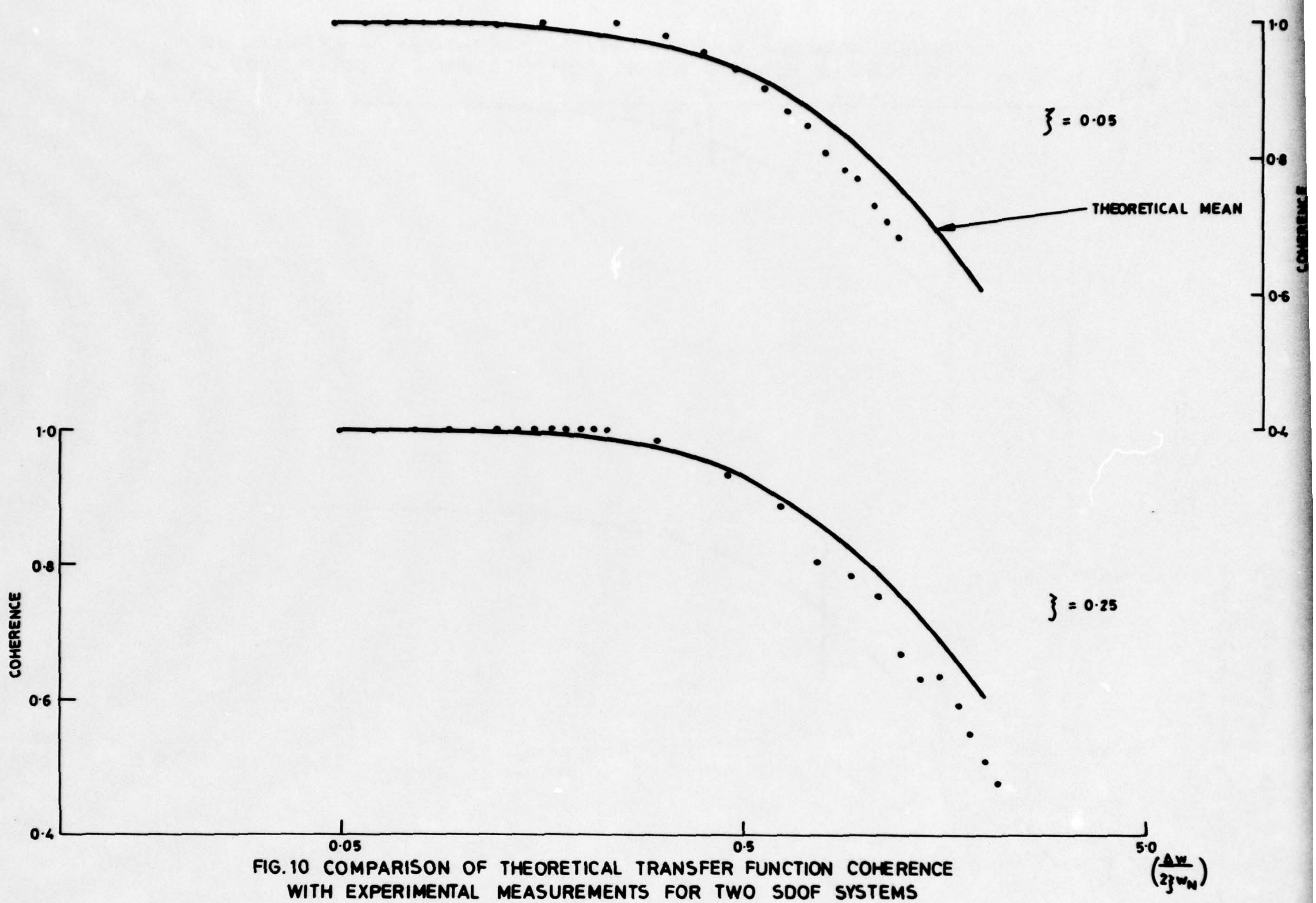


FIG. 10 COMPARISON OF THEORETICAL TRANSFER FUNCTION COHERENCE WITH EXPERIMENTAL MEASUREMENTS FOR TWO SDOF SYSTEMS

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